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ABSTRACTS

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LIST OF TALKS

Olusola John Adeniran

On the Crypto-automorphism of the Buchsteiner Loops

In this study, new identities of Buchsteiner loops were obtained via the principal isotopes. It was also shown that the middle inner mapping T_v^{-1} is a crypto-automorphism with companions v and v^{λ}. Our results which are new in a way, complement and extend existing results in the literature

Marcin Anholcer

Irregular Labelings of Circulant Graphs

We investigate the *irregularity strength* ($s(G)$) and *total vertex irregularity strength* ($tvs(G)$) of circulant graphs $Ci_n(1, 2, \ldots, k)$.

We prove that $tvs(Ci_n(1, 2, \ldots, k)) = \frac{n+2k}{2k+1}$, while $s(Ci_n(1, 2, \ldots, k)) = \frac{n+2k-1}{2k}$. In order to do that, we split the graph $Ci_n(1, 2, \ldots, k)$ into segments and label each segment using 0, 1 and 2 in such a way that the weighted degrees of the vertices included in that segment are distinct multiplicities of 2. In the next step we multiply all the edge labels by about $s/2$ (depending on the parity of s) in order to obtain the labeling where all the weighted degrees in any chosen segment differ by at least s. Then by changing the weighted degrees in every segment by distinct integer from the set $\{1, 2, \ldots, s\}$ we obtain the desired irregular weighting.

Asmiati

The Locating-Chromatic Number of Non Homogeneous Firecracker Graphs

Let G be a connected graph. Let c be a proper k-coloring of G, namely $c(u) \neq c(v)$ for any adjacent vertices u and v in G. For $i = 1, 2, \ldots, k$ define the color class C_i as the set of vertices receiving color i. The color code $c_{\Pi}(v)$ of a vertex v in G is the ordered k-tuple $(d(v, C_1), d(v, C_2), \ldots, d(v, C_k))$ where $d(v, C_i) = \min\{d(v, x) | x \in C_i\}$ for any i. If all distinct vertices of G have distinct color codes, then c is called a locating-coloring of G. The locating-chromatic number of graph G, denoted by $\chi_L(G)$ is the smallest k such that G has a locating coloring with k colors. A non homogeneous firecracker graph $F_{n,(k_1,k_2,\ldots,k_n)}$ is a graph obtained by concatenating n stars, each of k_i vertices, $k_i \geq 2$, by linking one leaf from each star. In this paper we determine the locating-chromatic number of non homogeneous firecracker graphs $F_{n,(k_1,k_2,\ldots,k_n)}$.

Mindaugas Bloznelis

Birth of a Strongly Connected Giant in an Inhomogeneous Random Digraph

Consider a model of inhomogeneous random digraphs with labelled vertices, where the arcs are generated independently, and the probability of inserting an arc depends on the labels of its endpoints and its orientation. For this model the critical point for the emergence of a giant component is determined via a branching process approach.

Bartłomiej Bosek

The sub-exponential Upper Bound for on-line Chain Partitioning

The main question in the on-line chain partitioning problem is to determine whether there exists an algorithm that partitions on-line posets of width at most w into polynomial number of chains - see Trotter's chapter Partially Ordered Sets in the Handbook of Combinatorics. So far the best known on-line algorithm of Kierstead used at most $(5^w - 1)/4$ chains; on the other hand Szemerédi proved that any on-line algorithm requires at least $\binom{w+1}{2}$ chains. These results were obtained in the early eighties and since then no progress in the general case has been done.

Together with Tomasz Krawczyk provide an on-line algorithm that partitions orders of width w into at most $w^{14 \log w}$ chains. This yields the first sub-exponential upper bound for on-line chain partitioning problem.

Michał Dębski

Decomposing 2-colored Boolean Lattice into 2-colored Chains of Length 2

We consider the following problem: Let L be a finite boolean lattice. Each of its elements is colored either blue or red, with the following restrictions: (a) an element x is blue if and only if $-x$ is red, and (b) if an element x is blue, than any element y such that $y < x$ is also blue. The question is whether there exists a decomposition of L into blue-red pairs. Precisely: can lattice L be decomposed into disjoint chains of length 2, each consisting of a blue and a red element? This question was originally formulated by P. Mazur in terms of products of prime numbers.

We give the solution to this problem.

Andrzej Dudek

Hamilton Cycles in Random Hypergraphs

In the random k-uniform hypergraph $H_{n,p;k}$ of order n each possible k-tuple appears independently with probability p. A loose Hamilton cycle is a cycle of order n in which every pair of adjacent edges intersects in a single vertex. In this talk we show that $(\log n)/n^{k-1}$ is the asymptotic threshold for the existence of loose Hamilton cycles in $H_{n,p;k}$.

This is joint work with Alan Frieze.

Anna Fiedorowicz

Ramsey-type Theorems Related to Q-colourings of Graphs

Suppose $\mathcal{Q}$ is a hereditary graph property and assume $\mathcal{Q} \subseteq \mathcal{O}^2$, where $\mathcal{O}^2$ denotes the class of bipartite graphs. We define a $(\mathcal{Q}, k)$-*colouring* of a graph G as a mapping $f : V(G) \to C$, where $C = \{1, \ldots, k\}$ is a set of *colours*, satisfying the condition that for every two distinct colours i and j, the subgraph induced in G by all the edges linking a vertex coloured with i and a vertex coloured with j belongs to $\mathcal{Q}$. If we additionally assume that for every colour i the set of vertices coloured with i is independent, then these $(\mathcal{Q}, k)$-colourings are a natural generalization of acyclic colourings if $\mathcal{Q}$ is the class of acyclic graphs, star colourings if $\mathcal{Q}$ is the class of star-forests, and so on.

Let $\mathcal{P}$ be a graph property and assume $k \geq 2$. We say that $\mathcal{P}$ is a $(\mathcal{Q}, k)$-*Ramsey class*, if for every $G \in \mathcal{P}$ there exists $H \in \mathcal{P}$ such that for every $(\mathcal{Q}, k)$-colouring of H there is a colour i such that $G \subseteq H[V_i]$, where V_i is the set of vertices coloured with i.

The notion of $(\mathcal{Q}, 2)$-Ramsey classes of graphs was introduced in [1], as a generalization of vertex-Ramsey classes of graphs, see [2] for a survey.

We prove that some well-known and important graph properties, such as k-degenerate graphs, k-trees and chordal graphs, are $(\mathcal{Q}, k)$-Ramsey classes of graphs. We also present sufficient conditions for a graph property to be a $(\mathcal{Q}, k)$-Ramsey class.

This is joint work with Mieczysław Borowiecki.

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Przemysław Gordinowicz

Let's Play the Cleaning Game

The cleaning game is a combinatorial game related to the model for cleaning graphs with brushes, studied, for example, by Alon, Messinger, Nowakowski, Prałat, and Wormald.

The board of the game is a fixed, finite, undirected graph G. At the beginning all the graph G is *dirty* (that is, all edges and vertices are dirty). Two players alternately put one brush on some vertex. At any point of the game, if there is a dirty vertex with at least as many brushes as incident dirty edges, the *cleaning process* starts: the vertex and all its incident edges are cleaned by moving exactly one brush along every dirty edge. The process continues until there is no dirty vertex with a sufficient number of brushes. The game finishes when all the graph is cleaned.

In the impartial game, the player who puts the last brush wins the game. For this variant, we show that the second player wins on every complete graph with at least 3 vertices. Other results and open problems are mentioned.

This is joint work with Paweł Prałat.

Katarzyna Górska

On the Combinatorial Content of One-sided Lévy Stable Probability Distributions

We report on recent findings of exact and explicit expressions for one-sided, heavy-tailed Lévy stable probability distributions $g_\alpha(x)$, $0 < x < \infty$, of index α, $0 < \alpha < 1$, for all $\alpha = l/k$, with k and l positive integers. We shall exemplify analytically and graphically several examples of known and new cases of such distributions. We point out that $g_{l/k}(x)$ is a solution of a *negative-power* Stieltjes type moment problem of the form $\int_0^\infty x^{-ln} g_{l/k}(x)dx = \frac{(kn)!}{(ln)!}$, $n = 0, 1, \ldots$, i.e. with negative moments being integer combinatorial sequences of factorial type. This last relation, when seen as a conventional Stieltjes moment problem, can be solved with the use of inverse Mellin transform. In this way we derive an explicit formulae for $g_{l/k}(x)$ in terms of Meijer G functions. The problem of non-uniqueness of so obtained solutions is briefly discussed.

This is joint work with Karol A. Penson.

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Mariusz Grech

Semiregular Automorphism Groups of Graphical Structures

By $GR(k)$, $DGR(k)$, $BGR(k)$, $SGR(k)$ we denote the classes of automorphism groups of k-edge colored graphs, digraphs, hypergraphs, and supergaphs of order k. The more general problem in these areas is: to describes all these classes for each k.

I show the solution of this problem for the case of semiregular permutation groups, i.e., for every semiregular permutation group A, I show k_1, k_2, k_3, k_4 such that $A \in GR(k_1) \setminus GR(k_1 - 1)$, $A \in DGR(k_2) \setminus DGR(k_2 - 1)$, $A \in BGR(k_3) \setminus BGR(k_3 - 1)$, and $A \in SGR(k_4) \setminus SGR(k_4 - 1)$.

Andrzej Grzesik

Indicated Colorings of Graphs

Let us consider a game in which two players color a graph. The first player (Indicator) selects a vertex and the second player (Painter) colors it in a proper way in one of available colors from a fixed set. The goal of Indicator is to color the whole graph and the goal of Painter is to spoil it. The smallest number of colors which is necessary for Indicator to win the game (regardless of the strategy of Painter) will be called indicated chromatic number. In the talk we will present a few facts about indicated chromatic number, in particular we will show some lower and upper bounds. We will also consider this number in some particular classes of graphs (for example planar graphs). We will also present a class of graphs for which the indicated chromatic number is equal to the chromatic number.

Pavol Hell

List Homomorphisms

List homomorphisms are a common generalization of homomorphisms and list colourings. The existence of homomorphisms to a fixed digraph is a difficult problem, at the core of the dichotomy conjecture of constraint satisfaction. The existence of list homomorphisms (and list constraint satisfaction problems) is much better behaved, and its dichotomy is known. For undirected graphs H, the list homomorphisms to H can be solved in polynomial time precisely for graphs with nice structure, say interval graphs in the reflexive case. For list constraint satisfaction problems, there is an algebraic classification based on the notion of polymorphism. I will survey the background and describe a new combinatorial classification of the complexity of list homomorphisms to digraphs, akin to those for undirected graphs.

This is joint work with Arash Rafiey.

Leszek Horwath

Preemptive Online Weighted Scheduling

Let us consider the following 2 player game. First player is preemptive scheduler, while the second one is buffer of jobs. Every i-th job requires to be completed continuously from time a_i to b_i and in case of completion second player will give c_i goods to the first one. In the talk I would speak about boundaries for competitive constant for online version of such problem.

Moharram N. Iradmusa

On Automorphisms Groups of Graph Powers

An automorphism of a graph G is a permutation π of the vertex set of G with the property that, for any vertices u and v of G, we have $\pi(u) \sim \pi(v)$ if and only if $u \sim v$. The set of all automorphisms of a graph G, with the operation of composition of permutations, is a permutation group on $V(G)$, denoted by $Aut(G)$. The automorphism group of a graph characterize its symmetries, and is therefore very useful in determining certain of its properties.

The $k-$*power of* G, denoted by G^k, is defined on the vertex set $V(G)$ by adding edges joining any two distinct vertices x and y with distance at most k. In other words, $E(G^k) = \{xy : 1 \leq d_G(x,y) \leq k\}$.

In this paper, we investigate the automorphism groups of the graph powers and show that $Aut(G^n) \cong Aut(G)$, provided that G is a connected graph with sufficiently large girth and without terminal vertices. In addition we show that $Aut(G^2) \cong Aut(G)$, when $diam(G) \geq 4$ and $g(G) \geq 7$.

Katarzyna Jesse-Józefczyk

Monotonicity and Expansion of Global Secure Sets

In the talk two problems will be presented, namely finding a global secure set of fixed cardinality and its expansion. For a given graph $G = (V, E)$ a global secure set $SD \subseteq V$ is a dominating set which also satisfies a condition that $|N[X] \cap SD| \geq |N[X] - SD|$ for every subset $X \subseteq SD$. Moreover we say that a global secure set $SD \subset V$ is expandable if there exists a vertex $v \in V - SD$ such that the set $SD' = SD \cup \{v\}$ is a global secure set.

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Konstanty Junosza-Szaniawski

On Improved Exact Algorithm for $L(2,1)$-labeling of Graphs

$L(2,1)$-labeling is a graph coloring model inspired by a channel assignment problem in telecommunication. It asks for such a labeling of vertices with nonnegative integers that adjacent vertices get labels that differ by at least 2 and vertices in distance 2 get different labels. It is proved by Fiala *et al.* [1] that for any fixed $k \geq 4$ it is NP-complete to determine if a graph has an $L(2,1)$-labeling with no label greater than k.

Havet *et al.* [1] presented an algorithm for finding an optimal $L(2,1)$-labeling (i.e. an $L(2,1)$-labeling in which the largest label is the least possible) of a graph whose time complexity is $O^*(3.8739^n)$.

In this talk we present and analyze an improved version of this algorithm. The time complexity bound $O^*(3.5616^n)$ of our algorithm is substantially better than the time complexity of the original one. The difference lies in a better bound on the number of 2-packings, a smaller number of triples considered in the main loop of the algorithm and more carefully formulated conditions for this loop.

Moreover space complexity of our algorithm is improved from $O^*(3^n)$ to $O(2.56^n)$.

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Rafał Kalinowski

Edge Distinguishing Index of a Graph

A neighbourhood of an edge of a simple graph is a subgraph induced by this edge and all edges adjacent to it. We say that a given colouring of edges distinguishes a pair of edges, if there does not exist an isomorphisms of their neighbourhoods preserving colours. The smallest number of colours in a colouring which distinguishes every pair of edges is called the edge distinguishing index of a graph.

We discuss the edge distinguishing index for some simple classes of graphs.

This is joint work with Mariusz Woźniak.

Wojciech Kaźmierczak

Best Choice Problem for Single-branching Binary Trees

We find optimal stopping times for the secretary problem considered on partially ordered sets whose Hasse diagrams are rooted single-branching binary trees. We also determine the optimal probability of success, i.e. choosing the best element, and the asymptotic value of this probability and a certain threshold behaviour of the optimal stopping times when the number of elements in our set tends to infinity.

Andrzej Kisielewicz

Supergraphs

Supergraphs are simple graphical structures extending graphs. Every permutation group is an automorphism group of a supergraph. This makes supergraphs a tool in investigation of complexity of permutation groups.

Mekkia Kouider

On f-factors

Let $f : X \to \mathbb{N}$ be an integer function on the set X. An f-factor of the graph $G = (X, E)$ is a spanning subgraph of G whose vertices have degrees defined by f. If $f \equiv 2$, we have a 2-factor. A family of vertex-disjoint cycles of G will be called a pseudo 2-factor. We present in this talk some sufficient conditions for the existence of an f-factor; these conditions involve the stability number, the minimum degree, or the connectivity of the graph.

On the other side, we shall speak about the complement of a maximum pseudo 2-factor.

This is joint work with Siham Bekkai.

Jakub Kozik

Generalized Secretary Problem - Dynamic Threshold Strategy

There are N candidates applying for a single secretary position. The candidates are interviewed in a random order. After each interview an irrevocable decision to accept or reject the current candidate is made. Once a candidate is accepted, no more interviews are arranged. All candidates are partially ordered according to their qualifications, but their positions, in the poset of candidates interviewed so far, are not known before the interview. The objective is to maximize the probability of accepting one of the best candidates. We are going to discuss a new strategy for the problem (Dynamic Threshold Strategy), which outperforms the best universal strategy known so far (Preater's strategy).

Tomasz Krawczyk

First-Fit Coloring of Cocomparability Graphs

One of the simplest heuristics for obtaining a chain partition of a poset is First-Fit algorithm. First-Fit labels chains with natural number and, for a given ordering of points of a poset, assigns to each point the lowest possible natural number.

First-Fit does not perform well in partitioning general posets into chains. Kierstead noted that on a class of posets of width 2 First-Fit uses unlimited number of chains. In this talk we show that the number of chains used by First-Fit on a class of H-free posets of width at most w is bounded if and only if H is a poset of width 2.

This is joint work with B. Bosek and G. Matecki.

Przemysław Krysztowiak

On Fixed-parameter Tractability of the Jump Number Problem of Interval Orders

The jump number problem of posets consists of determining a linear extension that minimizes the number of noncomparable adjacent elements. This problem has applications in the area of one-processor scheduling, where a class of posets called interval orders is applicable, i.e., posets in which the order relation is given by placement of intervals on the real line. Let us denote the jump number of given poset P by $s(P)$. It is NP-complete to determine $s(P)$ both in the general case of arbitrary posets [1], as well as if only interval orders are considered [3]. We are interested in answering the following question: is $s(P)$ less than or equal to k, where k is a fixed parameter? This question has been answered by means of a fixed-parameter algorithm for arbitrary posets [2]. We demonstrate our latest results concerning the fixed parameter tractability of determining $s(P)$ in a class of interval orders.

References

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Krzysztof Krzywdziński

The Complexity of Distributed BFS in ad hoc Networks

In the talk we are going to study time and message complexity of the problem of building a BFS tree in ad hoc network by a spontaneously awaken node. In the model computation is in synchronous rounds, and messages are sent via point-to-point bi-directional links. Network topology is modeled by an undirected graph. Each node knows only its own id and the id's of its neighbors in the network. We are going to present a deterministic algorithm that trades time for messages, mainly, with time complexity $O(\mathrm{diam} \cdot \min(\mathrm{diam}, n/f(n)) \cdot \log \mathrm{diam} \cdot \log n)$ and with the number of point-to-point messages sent $O(n \cdot (\min(\mathrm{diam}, n/f(n)) + f(n)) \cdot \log \mathrm{diam} \cdot \log n)$, for any monotonically non-decreasing sub-linear integer function f. This yields the first BFS-finding deterministic algorithm in ad hoc network working in time $o(n)$ and with $o(n^2)$ message complexity, for suitable function $f(n) = \omega(\mathrm{diam} \log^2 n)$, provided $\mathrm{diam} = o(n/\log^2 n)$.

This is joint work with Dariusz Kowalski.

Małgorzata Kuchta

Multiple Secretary Problem

We consider two secretary type problems where a selector who has only one choice is to choose a maximal element from a set that is a union of m linearly ordered sets. In the first problem (*with many lives*) the candidates from the first linearly ordered set are examined and if no candidate is chosen then the candidates from the second set are examined etc. In the second (*cyclic*) problem the first candidate comes from the first set and every next candidate comes from the next set and if no choice is made the search returns to the first set etc. In both cases an optimal stopping time and the probability of its success are found as well as their asymptotic behaviour.

This is joint work with Michał Morayne

Valentas Kurauskas

Small Subgraphs in Random Intersection Digraphs

Given subsets $S^-(1), \ldots, S^-(n)$ and $S^+(1), \ldots, S^+(n)$ of a set $W = \{w_1, \ldots, w_m\}$, define an intersection digraph on the vertex set $V = \{v_1, \ldots, v_n\}$ by putting an arc $v_i v_j$ for $i \neq j$ if and only if the sets $S^-(v_i)$ and $S^+(v_j)$ share a common element. Assuming the sets $S^-(i), S^+(i)$, $i = 1, \ldots, n$ are drawn at random, we obtain a random intersection digraph. Assuming further that for any $i = 1, \ldots, n$ and $j = 1, \ldots, m$ we have $P(w_j \in S^-(i)) = p_-(n, m)$ and $P(w_j \in S^+(i)) = p_+(n, m)$ and each inclusion is independent we obtain a binomial random intersection digraph $G(n, m, p_-, p_+)$.

This (and more general) random intersection digraph model first studied by Bloznelis [1] is a natural extension of the random intersection graph introduced by Karoński, Scheinerman and Singer-Cohen [3]. The latter paper considers the problem of appearance of constant-size induced subgraphs H in the binomial random intersection graph $G(n, m, p)$ and obtains the explicit thresholds for appearance and disappearance for various types of small induced subgraphs H such as $H = K_h$, the complete graph on h vertices.

In the present work we consider appearance thresholds for small subgraphs of the binomial random intersection digraph $G(n, m, p_-, p_+)$ and determine explicit functions that characterize the appearance threshold for $\overrightarrow{K}_h$, the digraph on h vertices with all possible arcs but without loops.

Identify each $w \in W$ with a unique colour. Then $G(n, m, p_-, p_+)$ can be treated as a random coloured directed multigraph. We obtain that in almost all cases one of four simple types of coloured digraphs $\overrightarrow{K}_h$ is born first ("monochromatic", "rainbow", "in-star" or "out-star") depending on the parameters m, p_- and p_+.

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Michał Lasoń

Extremal Problem for Crossing Vectors

Vectors v and u in the lattice Z^w i-cross if there exists a coordinate on which v is i bigger then u and vice versa. The talk is devoted to the following problem: What is the maximal cardinality of a set of vectors that pairwise 1-cross, but not k-cross? This problem has some order theoretic motivation which will be explained. Our conjecture is that the answer is $k^{(}w - 1)$. It is true for $w = 1, 2$ and 3. We will show several examples realizing expected number and show proofs of two incomparable upper bounds k^w and $wk^{(}w-1)$.

Shabnam Malik

Hamiltonian Properties of Directed Toeplitz Graph

An $(n \times n)$ matrix $A = (a_{ij})$ is called a Toeplitz matrix if it has constant values along all diagonals parallel to the main diagonal. A directed Toeplitz graph is a digraph with Toeplitz adjacency matrix. In this talk I will discuss conditions for the existence of hamiltonian cycles in directed Toeplitz graphs.

I will discuss the hamiltonicity of the Toeplitz graphs of type $T_n\langle 1, 3, 4; t\rangle$. For $t \in \{2, 3, 4, 5, 8, 9\}$, condition will be discussed (on n) under which such a graph is hamiltonian. For $t \in \{6, 7\}$ and $t \geq 10$, we will see that $T_n\langle 1, 3, 4; t\rangle$ is hamiltonian for all n.

Grzegorz Matecki

Adaptive Matching on Bipartite Graphs

We consider bipartite matching in the on-line version as follows. There is a bipartite graph $G = (U, V, E)$, in which vertices in V are given a priori and each vertex u in U arrives in the on-line fashion (with all its incident edges). An on-line algorithm matches each such vertex u to a previously unmatched adjacent vertex in V, if there is one. Decisions made by the algorithm are irrevocable. The objective is to maximize the size of the resulting matching. It is easy to observe that any greedy algorithm (never leave vertex u unmatched if a match is possible) matches at least $n/2$ edges where n is the size of the optimal matching with G given at once. This number is optimal and there is no better deterministic algorithm.

We propose the following deterministic modification of an on-line matching. The algorithm matches each incoming vertex u in U to a set $S(u)$ of adjacent vertices in V (instead of one vertex). In case when $S(u)$ and $S(x)$ for already existing x in U are not disjoint the algorithm must remove all common vertices from $S(x)$. Additionally, the algorithm has to obey the rule: each $S(x)$ must not become empty if only it was initialized with a nonempty set of vertices. An algorithm satisfying the above condition is called adaptive. In this approach a vertex u in U can be always matched to a vertex from $S(u)$ (if $S(u)$ is not empty). Therefore, the number of matched edges is equal to the number of nonempty sets $S(u)$. We are going to present the optimal adaptive on-line algorithm which breaks $n/2$ barrier and matches at least $(1 - \pi/\cosh(\sqrt{3}\pi/2))n + o(n) = 0.589n + o(n)$ edges.

This is joint work with Jakub Kozik.

Angela Mestre

Riordan Arrays via Umbral Calculus

We use the classical umbral calculus to define the Riordan group. We introduce a new Riordan subgroup which we call extended Bell subgroup. We give umbral expressions for the elements of the Appell, associated, Bell and extended Bell subgroups. To this end, we derive several useful identities which involve the primitive or derivative of the composition umbra. Our method consists of an adequate manipulation of exponential generating functions of umbrae. This is also used to derive two umbral versions of the Lagrange inversion formula recently introduced in the literature. Furthermore, we study the effects of several transformation rules on the entries of a Riordan array when regarded as functions of two integer variables.

This is joint work with José Agapito and Maria Manuel Torres.

Piotr Micek

Nonrepetitive Sequences

A repetition of size h ($h \geq 1$) in a given sequence is a subsequence of consecutive terms of the form: $xx = x_1, \ldots, x_h, x_1, \ldots, x_h$. A sequence is nonrepetitive if it does not contain a repetition of any size. The remarkable construction of Thue asserts that 3 different symbols are enough to build an arbitrarily long nonrepetitive sequence. A sequence $s_1, \ldots, s_n$ is chosen from lists (sets) $L_1, \ldots, L_n$ if $s_i \in L_i$ for all i. It is still not settled what is the minimum size of all the lists required to ensure that there is a nonrepetitive sequence chosen from them. We present a very simple proof that lists of size 4 suffice (equalizing the best known bound) and provide a natural randomized algorithm constructing such a sequence in expected linear time. Our approach is inspired by a new algorithmic proof of Lovász local lemma due to Moser and Tardos and previous work of Moser (his so called entropy compression argument). We also consider game-theoretic versions of results on nonrepetitive sequences. A nonrepetitive game is played by two players who pick, one by one, consecutive terms of a sequence over a given set of symbols. First player tries to avoid repetitions while second one does not need to cooperate. Of course, by simple imitating the second player can force lots of repetitions of size 1. Pegden proved that there is a strategy for the first player to build an arbitrarily long sequence with no repetitions of size greater than 1 over 37 symbols. Our techniques allow to reduce 37 to 6. The second game we consider is an erase-repetition game. Here, whenever a repetition occurs the repeated block is immediately erased and the next player to play continues. We prove that there is a strategy for first player to build an arbitrarily long nonrepetitive sequence over 8 symbols.

This is joint work with Jarosław Grytczuk and Jakub Kozik.

Martin Nehéz

A Detailed Study of the Dominating Cliques Phase Transition in Random Graphs

A subset of nodes $S \subseteq V$ of a graph $G = (V, E)$ is a dominating clique if S is a dominating set and a clique of G. The phase transition of dominating cliques in Erdős-Rényi random graph model $G(n,p)$ is investigated in this paper. Lower and upper bounds on the edge probability p for the existence of an r-node dominating clique are established in this paper. We prove therein that given an n-node random graph G from $G(n,p)$ for $r = c\log_{1/p} n$ with $1 \leq c \leq 2$ it holds: (1) if $p > 1/2$ then an r-clique is dominating in G with a high probability and, (2) if $p \leq (3-\sqrt{5})/2$ then an r-clique is not dominating in G with a high probability. The remaining range of the probability p is discussed with more attention. Within such a range, we provide intervals of r where a dominating clique existence probability is zero, positive but less than one, and one, respectively.

Karol A. Penson

Fuss-Catalan and Raney Distributions Versus Products of Random Matrices

We explicitly find positive measures $P_s(x)$ whose n-th Hausdorff power moment is the sequence of Fuss-Catalan numbers, defined by $FC_s(n) = \frac{1}{sn+1}\binom{sn+n}{n}$, with $s = 1, 2, \ldots$ and $n = 0, 1, \ldots$ Two-parameter generalization of Fuss-Catalan numbers, the Raney numbers are defined by $R_{r,k}(n) = \frac{k}{rn+k}\binom{rn+k}{n}$, $r = 1, 2, \ldots$, $k = 1, 2, \ldots$ and $n = 0, 1, \ldots$ We explicitly find positive measures $W_{r,k}(x)$ whose n-th Hausdorff moment is the sequence $R_{r,k}(n)$. We discuss analytically and graphically these measures and demonstrate that $P_s(x)$ generalize the Marchenko-Pastur distribution, and $W_{r,k}(x)$ is a natural extension of Wigner's semicircle law, both characterizing different forms of products of random matrices.

This is joint work with Karol Życzkowski.

Nataliya Petryshyn

On Rooted Packings of Graphs

A *vertex H-packing* (respectively an *edge H-packing*) into a graph G is a collection of vertex disjoint (resp. edge disjoint) subgraphs of G, each isomorphic to H. By a *rooted graph* we mean a pair (H, S), where H is a graph and $S \subseteq V(H)$. Rooted graphs (G, T) and (H, S) are *isomorphic* if there is a bijection $\varphi : V(G) \to V(H)$ such that $xy \in E(G)$ iff $\varphi(x)\varphi(y) \in E(H)$ and $\varphi(T) = S$. A collection of rooted graphs $\{(H_1, S_1), \dots, (H_k, S_k)\}$ isomorphic to (H, S) is a *rooted (H, S)-packing* into a graph G if $H_1, \dots, H_k$ are subgraphs of G, the edge sets $E(H_1), \dots, E(H_k)$ are pairwise disjoint and the vertex sets $S_1, \dots, S_k$ are pairwise disjoint. The concept of a rooted (H, S)-packing is a common generalization of both a vertex H-packing and an edge H-packing into a graph. We denote by $r(G, H, S)$ the largest number of copies of (H, S) in a rooted (H, S)-packing into G. A rooted (H, S)-packing into a graph G is a *rooted (H, S)-decomposition* (resp. a *rooted (H, S)-factor*) if $E(H_1) \cup \ldots \cup E(H_k) = E(G)$ (resp. $S_1 \cup \ldots \cup S_k = V(G)$).
In the presented work we give a characterization of graphs that have a rooted (H, S)-decomposition and/or a rooted (H, S)-factor, for $H = K_{1,2}$ and some sets $S \subseteq V(H)$. In these cases we also compute the number $r(G, H, S)$. Next we give some negative computational complexity results on the problem of existence of a rooted (H, S)-decomposition, for some specific graphs H and sets $S \subseteq V(H)$.

Irmantas Radavičius

On Existence of Hamilton Cycles in Uniform Random Intersection Graphs

We give a sufficient condition for the hamiltonicity of the uniform random intersection graph $G_{n,m,d}$. It is a graph on n vertices, where each vertex is assigned d keys drawn independently at random from a given set of m keys, and where any two vertices establish an edge whenever they share at least one common key. We show that with probability tending to 1 the graph $G_{n,m,d}$ has a Hamilton cycle provided that $n = 2^{-1}m\big(\ln m + \ln\ln m + \omega(m)\big)$ with some $\omega(m) \to +\infty$ as $m \to \infty$.

Ian Roberts

Extremal problems in Sperner Theory and Combinatorial Designs

Completely separating systems and antichains are dual structures. Some recent work, and a range of open problems in either Combinatorial Design Theory or Sperner Theory, will be considered.

Jolanta Rosiak

LOCAL EXPONENTS OF PRIMITIVE DIGRAPHS

A digraph D is primitive if there exists an integer $t > 0$ such that for all ordered pairs of vertices u and v (not necessarily distinct) there is a directed walk from u to v of length t in D. The smallest such t is called the exponent of D, denoted $\exp(D)$. If D is primitive and $u \in V(D)$, then there exists an integer $t > 0$ such that for each $v \in V(D)$ there exists a directed walk from u do v in D of length t. The smallest such t, denoted $\exp_D(u)$, is called the exponent of the vertex u in D (or the local exponent of D at the vertex u). Then $\exp(D) = \max_{u \in V(D)} \{\exp_D(u)\}$.

If D is a primitive digraph and $|V(D)| = n$, then the vertices can be relabeled as $v_1, v_2, \dots, v_n$ so that $\exp_D(v_1) \leq \dots \leq \exp_D(v_n)$. Let $\varepsilon(D) = \exp_D(v_n) - \exp_D(v_1)$. The problem of characterizing the sets $L^o_{n,m}(k) = \{\exp(D) : D \in \mathcal{DP}^o(n,m) \wedge \varepsilon(D) = k\}$, where $\mathcal{DP}^o(n,m)$ is the class of primitive digraphs with n vertices containing loops exactly at m vertices, is considered.

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Łukasz Rożej

Decomposition of Sparse Claw-free Graphs, with Application to Game Coloring

A recent result by Montassier et al. studies decomposition of sparse graphs with its application to graph coloring. We investigate how the result can be improved for claw-free graphs. Let $f(k)$ be the least number such that every claw-free graph with maximum average degree less then $f(k)$ is decomposable into a tree and a graph of maximum degree at most k. We give lower and upper bounds of $f(g)$.

Katarzyna Rybarczyk

A New Method for Establishing Sharp Threshold Functions in Random Intersection Graphs

In a random intersection graph $G(n,m,p)$ to each vertex v from a vertex set V we assign a set of its features D_v by choosing independently each feature with probability p from a feature set W. Then we connect vertices $v, v' \in V$ by an edge if and only if sets D_v and $D_{v'}$ intersect. In the talk a new method for establishing threshold functions in $G(n,m,p)$ will be presented. It will be used to determine sharp threshold functions in $G(n,m,p)$ for k–connectivity, perfect matching containment and Hamilton cycle containment. In fact it will be shown that in some cases it is possible to obtain interesting results using relations between $G(n,m,p)$ and random graph with independent edges, despite the fact that two models differ by a lot.

Wojciech Rytter

The Structure of Graphs Representing All Subwords of Thue-Morse Sequences

The directed acyclic subword graphs are a useful tool as data structures representing succinctly all subwords of a given word. For certain families of words (Fibonacci, Sturmian, Thue-Morse) the compacted versions of these graphs are of logarithmic size with respect to the size of the input word and have very regular structure. These compacted graphs are of independent interest as combinatorial objects. We discuss several structural properties of the graphs related to finite and infinite Thue-Morse sequences.

Paweł Rzążewski

On the Number of 2-packings in a Connected Graph

There are some famous results in extremal graph theory e.g. on number of maximal clicks in a graph by Moon and Moser [2] or on number of maximal independent sets in a tree by Sagan [3]. In this talk we investigate another interesting problem in extremal graph theory, which was first researched by Havet, Klazar, Kratochvíl, Kratsch and Liedloff [1] in analysis of algorithms for $L(2,1)$-labeling.

A 2-packing is a subset of vertices of a graph, such that no two vertices from this set have a common neighbor. In this talk we discuss the maximum number of 2-packings in a connected graph.

We present the algorithm for generating all 2-packings of a specified size in a connected graph. An analysis of this algorithm provides a new upper bound on the maximum number of such 2-packings, which is $\binom{n-k+1}{k}$, where k denotes a cardinality of each generated 2-packing and n denotes a number of vertices in a graph.

Then we improve our method to generate all 2-packings in a connected graph and obtain a new upper bound on their number – $O^*(1.5399\ldots^n)$. Additionally, we present a lower bound on the maximum number of 2-packings, which is $\Omega^*(1.4977\ldots^n)$.

The application of these results in analysis of algorithms for L(2,1)-labeling will be presented in the the talk *On Improved Exact Algorithm for $L(2,1)$-labeling of Graphs* by Konstanty Junosza-Szaniawski.

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Wojciech Samotij

Large Bounded Degree Trees in Random Graphs

Recently, there have been a few developments in the study of embedding of large trees with bounded degree in random graphs. In this talk, we will give a quick survey of this area of research, focusing on the problem of existence of almost spanning trees with bounded degree in very sparse random graphs.

This is joint work with József Balogh, Béla Csaba, and Martin Pei.

Suhadi Wido Saputro

The Metric Dimension of the Composition Product of Graphs

A set of vertices W resolves a graph G if every vertex is uniquely determined by its coordinate of distances to the vertices in W. The minimum cardinality of a resolving set of G is called the metric dimension of G. In this paper, we consider a graph which is obtained by the composition product between two graphs. The composition product of graph G and H, which is denoted by $G[H]$, is the graph with vertex set $V(G) \times V(H) = \{(a, v)|a \in V(G), v \in V(H)\}$, where (a, v) adjacent with (b, w) whenever $ab \in E(G)$, or $a = b$ and $vw \in E(H)$. We give the general bounds of the metric dimension of a composition product of any connected graph G and H. We also show that the bounds are sharp.

Elżbieta Sidorowicz

Colouring Games and the Game Defect of Graphs

Let G be a graph and $(\mathcal{P}_1, \mathcal{P}_2, \ldots, \mathcal{P}_r)$ be an ordered set of additive hereditary properties. A $(\mathcal{P}_1, \mathcal{P}_2, \ldots, \mathcal{P}_r)$-game (a generalized colouring game) is defined as follows. The players take turns colouring G with colours from $\{1, \ldots, r\}$ such that for each $i = 1, 2, \ldots, r$ the induced subgraph $G[V_i]$ (V_i is the set of vertices of G with colour i) has the property $\mathcal{P}_i$ after each move of the players. If after $|V(G)|$ moves all vertices of the graph G are coloured, then Alice wins. If $\mathcal{P}_1 = \mathcal{P}_2 = \cdots = \mathcal{P}_r = \mathcal{P}$, then the $(\mathcal{P}_1, \mathcal{P}_2, \ldots, \mathcal{P}_r)$-game is called the $\mathcal{P}$-game. If $\mathcal{P}$ is a set of totally disconnected graphs, then the $\mathcal{P}$-game is the colouring game. If $\mathcal{P}$ is a set of graphs with maximum degree d, then the $\mathcal{P}$-game is the d-relaxed colouring game. We consider the $\mathcal{P}$-game when $\mathcal{P}$ is one of the following additive hereditary properties:
$\mathcal{O}_k = \{G : |V(G)| \leq k + 1\}$,
$\mathcal{S}_k = \{G : \Delta(G) \leq k\}$,
$\mathcal{D}_k = \{G : G \text{ is } k\text{-degenerated graph}\}$.
We discuss relations between the $\mathcal{P}$-game, the marking game and the edge version of the d-relaxed colouring game.

The least d such that Alice has a winning strategy on G for the d-relaxed colouring game with r colours is called the relaxed game defect of G and is denoted by $\mathrm{def}_g(G, r)$. By $\mathrm{def}_g^{\mathcal{O}}(G, r)$ we denote the least k such that Alice has a winning strategy on G for the $\mathcal{O}_k$-game with r colours. We determine the numbers $\mathrm{def}_g(G, r)$ and $\mathrm{def}_g^{\mathcal{O}}(G, r)$ for some classes of graphs.

Maria Emília Silva

A Generalization of Chromatic Polynomial of a Graph Subdivision

Considering the partitions of a set into nonempty subsets, an expression for the number of all partitions of a given type is deduced. The chromatic polynomials of a graph subdivision is generalized, considering two sets of colours, and a general explicit expression is obtained for this generalization. Using the above referred results, the generalized chromatic polynomial is determined for the particular case of complete graph subdivision.

This is joint work with Domingos M. Cardoso and Jerzy Szymański.

Karolina Sołtys

Kropki are PSPACE-complete

In this talk I would like to study the computational complexity of determining the winner in a given game of Kropki, a paper and pencil game popular in Central and Eastern European countries, sharing some similarities with the game of Go. I will prove that the derived decision problem is PSPACE-complete, basing on the analogous results on the complexity of Go. My proof involves merging approaches of two papers on Go and adding a simple combinatorial gadget to make for the differences between the two games.

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Małgorzata Sulkowska

Graph-theoretic Generalization of the Best Choice Problem; Randomized Analysis of a Simple Effective Algorithm for k-ary Trees

We consider the following on-line decision problem. The vertices of a graph which is a complete rooted directed k-ary tree are being observed one by one in some random order by a selector. At time t the selector examines the t-th vertex and knows the graph induced by the t vertices that have already been examined. The selector's aim is to choose the root by taking the currently examined vertex. We propose and analyze a simple deterministic algorithm for the selector to follow. Using randomized techniques it is shown that its probability $p(k)$ of the right choice tends to 1 as k tends to infinity. A multiple randomization is introduced and applied to find the asymptotic probability of success of this algorithm. It is shown that for the binary tree this asymptotic probability is equal to $2\ln(2) - 1$ and for the ternary tree $1.5\ln(3) - 2 + \pi/(2 \cdot 3^{0.5})$ with the height of the tree tending to infinity.

Ioannis Tasoulas

Statistics on Ordered Trees and Dyck Paths

Ordered trees statistics as well as Dyck paths statistics have been studied by many authors.

In this work, by considering bijections between ordered trees and Dyck paths, we deduce some known as well as some new equidistributions between them.

Mirosław Truszczyński

Representing and Reasoning about Preferences - Combinatorics Meets Decision Theory

The problems of eliciting, representing and computing with preferences over a multi-attribute domain arise in many fields such as planning, design, and group decision making. An explicit representation of a preference ordering is exponentially large in the number of attributes used to describe domain elements, typically referred to as outcomes or configurations. Therefore, AI researchers have developed languages for representing preference orderings multi-attribute domains in a succinct way, with the language of CP-nets being one of the most widely studied ones. It provides an elegant graphical model of representing preferences that implies an effective approach to preference elicitation and, at least in some cases, supports fast reasoning algorithms. In the talk, I will introduce CP-nets, define the fundamental reasoning problems of deciding consistency, dominance and optimality, and discuss their complexity.

Panagiotis-Georgios Tsikouras

The Enumeration of Strings at Height j in Dyck Paths

This work deals with (lattice) paths in the integer plane, consisting of two steps, rises and falls. An important class of such paths are the Dyck paths, which start and end at the same height and lie weakly above this height. A wide range of papers dealing with the occurrences of a given path τ, called in this context *string*, in a Dyck path appear frequently in the literature. All of them deal with particular strings, whereas recent results by the authors concerning arbitrary strings have been announced in the 7th Lattice Path Combinatorics Conference (2010).

A related problem is the enumeration of Dyck paths according to the occurrences of τ at height j.

This problem has been studied for particular strings when $j = 0$, by many authors. For positive j and $\tau = ud$ or $\tau = du$, Mansour (2002) expressed the generating function F_j which counts the number of occurrences of τ at height j via the Chebyshev polynomials of the second kind and the Catalan generating function. This idea was extended by A. Sapounakis, I. Tasoulas and the author (2007) for an arbitrary string τ, where F_j is expressed via the Chebyshev polynomials of the second kind and the generating function F_0. This result was then applied to every string of length 4, by evaluating F_0 (and hence F_j, too).

In this work, for an arbitrary string τ the generating function F_j is evaluated directly using a different method, based on the main characteristics of τ, namely number of rises, height, depth and periodicity.

This is joint work with K. Manes, A. Sapounakis and I. Tasoulas.

Michał Tuczyński

COUNTING TRANSVERSALS IN CLAW-FREE GRAPHS

In 2005 Dahllöf, Jonsson and Wahlström gave an algorithm that counts the number of all transversals in a graph on n vertices in time $O^*(1.25..^n)$. Their algorithm was improved by Fürer and Kasiviswanathan. The time complexity of their algorithm is $O^*(1.24..^n)$. We present an algorithm for claw-free graphs. It is based on construction of a cut-set such that components of a graph obtained by removing this cut-set are small compared to its order. We combine the cut-set strategy with methods of Dahllöf and others. As a result we obtain an algorithm which counts all transversals of a claw-free graph in time $O^*(1.23..^n)$.

This is joint work with Konstanty Junosza-Szaniawski.

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Torsten Ueckerdt

Enumeration of a Poset's Ideals

A subset of elements of a partially ordered set, poset for short, is an ideal, or downset, if it is closed under taking predecessors. Usually posets have exponentially many ideals and their enumeration has been long considered. So far the best known algorithm due to Habib, Medina, Nourine and Steiner takes time $O(\Delta)$ amortized per ideal. Here, Δ denotes the maximum number of immediate successors of an element.

In this talk we take a refined look at their algorithm, extracting optimal enumeration for 2-dimensional and planar posets.

Maciej Ulas

Arithmetic Properties of Certain Generalizations of Stern Diatomic Sequence

Let k be a positive integer. We study arithmetic properties of the sequence $\{f_k(n)\}$ given by the recurrence relation:

$$f_k(kn+i) = f_k(n+i) \quad \text{for} \quad i = 0, 1, \ldots, k-2,$$

and

$$f_k(kn+k-1) = \sum_{i=0}^{k-1} f_k(n+i).$$

The sequence $f_k(n)$ for $k \geq 3$ is a natural generalization of Stern diatomic sequence $f_2(n)$.

Bartosz Walczak

On-line Coloring of Rectangle Graphs

On-line graph coloring problems are variants of usual graph coloring problems in which the graph is not known in prior – its vertices are introduced one by one and the coloring algorithm has to color them immediately and irrevocably. Our interest lies in algorithms that find a proper coloring using as few colors as possible, regardless of the particular graph and its order of presentation. The competitiveness of algorithms is measured in terms of the off-line optimum (chromatic number) and the number of vertices, or, if more appropriate, in terms of the clique number w and the number of vertices.

We restrict our attention to intersection graphs of geometric objects and, moreover, we assume they are presented together with their geometric representations. Such a restriction arises naturally when considering practical applications of on-line coloring.

In one dimension, for interval graphs, the problem is well studied – the optimal on-line coloring algorithm uses $3w - 2$ colors in the worst case [1]. In two dimensions, it is known that First-Fit uses $O(w \log n)$ colors for intersection graphs of homothetic copies of any fixed convex shape (e.g. discs, axis-parallel squares). This bound follows from the bound of $O(w)$ for the coloring number of such graphs [2, 3], and it is asymptotically tight. Probably the simplest class of graphs that cannot be settled this way are the rectangle graphs – intersection graphs of axis-parallel rectangles in the plane. Here the number of colors used by First-Fit can be linear in n. We present an algorithm that colors rectangles on-line using $O(w^3 \log n)$ colors. For rectangles introduces in increasing order of the x-coordinates of their right edges we present an algorithm using $O(w \log n)$ colors, which is asymptotically tight.

This is joint work with Przemysław Mazur and Piotr Micek.

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Maciej Zwierzchowski

On Signed Star Domination Number

All graphs considered here are finite, undirected without loops and multiple edges. For a vertex $v \in V(G)$, $E_G(v) = \{uv : u \in V(G)\}$ is called the *edge-neighbourhood* of v in G and $\deg_G(v) = |E_G(v)|$ is called a *degree* of v in G. A vertex $v \in V(G)$ is an *isolated* vertex of G if $\deg_G(v) = 0$. Additionally we assume that G has no isolated vertex.

A function $f : E(G) \to \{-1, 1\}$ is a *signed star domination function* (*SSDF*) of G if $\sum_{e \in E_G(v)} f(e) \geq 1$ for every $v \in V(G)$.

Let $E_f^+(G) = \{e \in E(G) : f(e) = 1\}$. A *SSDF* f is called a *minimal signed star domination function* (*mSSDF*) of G if for every edge $e \in E_f^+(G)$, a function $g : E(G) \to \{-1, 1\}$ such that

$$g(e') = \begin{cases} f(e') & \text{if } e' \neq e \\ -1 & \text{if } e' = e \end{cases}.$$

is not a SSDF of G.

For a vertex $v \in V(G)$ by a symbol $f(v)$ we mean $\sum_{e \in E_G(v)} f(e)$. A sequence $\pi = (w_1, \ldots, w_n)$ of positive integers is called a **signed star domination sequence** if there exists a graph G with $V(G) = \{v_1, \ldots, v_n\}$ and a minimal signed star domination function f of G such that $f(v_i) = w_i$. We present a necessary and sufficient conditions for a sequence π to be a signed star domination sequence.

Wiktor Żelazny

Additive Colourings of Graphs

The additive colouring problem is vertex version of the "1-2-3 conjecture" proposed by Karoński, Łuczak, and Thomason. In this problem the vertices of simple finite graph G are coloured with elements of an abelian additive group Γ, then we define weight of vertex as sum of colours of its neighbours. If weights of every two adjacent vertices are different we say that G is Γ-colourable. The main question we're trying to answer is whether there is an upper bound on the minimal order of group that can be used to colour graph of bounded chromatic number? In my talk I'll present questions and partial results concerning additive colourings of interesting graph classes - including bipartite, planar, product and Cayley graphs.

This is joint work with Jarosław Grytczuk and Sebastian Czerwiński.