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Ramsey-type Theorems Related to Q-colourings of Graphs

Suppose $\mathcal{Q}$ is a hereditary graph property and assume $\mathcal{Q} \subseteq \mathcal{O}^2$, where $\mathcal{O}^2$ denotes the class of bipartite graphs. We define a $(\mathcal{Q}, k)$-*colouring* of a graph G as a mapping $f : V(G) \to C$, where $C = \{1, \ldots, k\}$ is a set of *colours*, satisfying the condition that for every two distinct colours i and j, the subgraph induced in G by all the edges linking a vertex coloured with i and a vertex coloured with j belongs to $\mathcal{Q}$. If we additionally assume that for every colour i the set of vertices coloured with i is independent, then these $(\mathcal{Q}, k)$-colourings are a natural generalization of acyclic colourings if $\mathcal{Q}$ is the class of acyclic graphs, star colourings if $\mathcal{Q}$ is the class of star-forests, and so on.

Let $\mathcal{P}$ be a graph property and assume $k \geq 2$. We say that $\mathcal{P}$ is a $(\mathcal{Q}, k)$-*Ramsey class*, if for every $G \in \mathcal{P}$ there exists $H \in \mathcal{P}$ such that for every $(\mathcal{Q}, k)$-colouring of H there is a colour i such that $G \subseteq H[V_i]$, where V_i is the set of vertices coloured with i.

The notion of $(\mathcal{Q}, 2)$-Ramsey classes of graphs was introduced in [1], as a generalization of vertex-Ramsey classes of graphs, see [2] for a survey.

We prove that some well-known and important graph properties, such as k-degenerate graphs, k-trees and chordal graphs, are $(\mathcal{Q}, k)$-Ramsey classes of graphs. We also present sufficient conditions for a graph property to be a $(\mathcal{Q}, k)$-Ramsey class.

This is joint work with Mieczysław Borowiecki.

References

[1] M. Borowiecki, A. Fiedorowicz, *On Partitions of Hereditary Properties of Graphs*, Discuss. Math. Graph Theory 26 (2006), 377–387.

[2] J. Nešetřil, *Ramsey Theory*, in: R.L. Graham, M. Grötschel, L. Lovász, eds., Handbook of Combinatorics, (North-Holland, 1995) 1331–1403.