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On-line Coloring of Rectangle Graphs

On-line graph coloring problems are variants of usual graph coloring problems in which the graph is not known in prior – its vertices are introduced one by one and the coloring algorithm has to color them immediately and irrevocably. Our interest lies in algorithms that find a proper coloring using as few colors as possible, regardless of the particular graph and its order of presentation. The competitiveness of algorithms is measured in terms of the off-line optimum (chromatic number) and the number of vertices, or, if more appropriate, in terms of the clique number w and the number of vertices.

We restrict our attention to intersection graphs of geometric objects and, moreover, we assume they are presented together with their geometric representations. Such a restriction arises naturally when considering practical applications of on-line coloring.

In one dimension, for interval graphs, the problem is well studied – the optimal on-line coloring algorithm uses $3w - 2$ colors in the worst case [1]. In two dimensions, it is known that First-Fit uses $O(w \log n)$ colors for intersection graphs of homothetic copies of any fixed convex shape (e.g. discs, axis-parallel squares). This bound follows from the bound of $O(w)$ for the coloring number of such graphs [2, 3], and it is asymptotically tight. Probably the simplest class of graphs that cannot be settled this way are the rectangle graphs – intersection graphs of axis-parallel rectangles in the plane. Here the number of colors used by First-Fit can be linear in n. We present an algorithm that colors rectangles on-line using $O(w^3 \log n)$ colors. For rectangles introduces in increasing order of the x-coordinates of their right edges we present an algorithm using $O(w \log n)$ colors, which is asymptotically tight.

This is joint work with Przemysław Mazur and Piotr Micek.

References

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