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Colouring Games and the Game Defect of Graphs

Let G be a graph and $(\mathcal{P}_1, \mathcal{P}_2, \ldots, \mathcal{P}_r)$ be an ordered set of additive hereditary properties. A $(\mathcal{P}_1, \mathcal{P}_2, \ldots, \mathcal{P}_r)$-game (a generalized colouring game) is defined as follows. The players take turns colouring G with colours from $\{1, \ldots, r\}$ such that for each $i = 1, 2, \ldots, r$ the induced subgraph $G[V_i]$ (V_i is the set of vertices of G with colour i) has the property $\mathcal{P}_i$ after each move of the players. If after $|V(G)|$ moves all vertices of the graph G are coloured, then Alice wins. If $\mathcal{P}_1 = \mathcal{P}_2 = \cdots = \mathcal{P}_r = \mathcal{P}$, then the $(\mathcal{P}_1, \mathcal{P}_2, \ldots, \mathcal{P}_r)$-game is called the $\mathcal{P}$-game. If $\mathcal{P}$ is a set of totally disconnected graphs, then the $\mathcal{P}$-game is the colouring game. If $\mathcal{P}$ is a set of graphs with maximum degree d, then the $\mathcal{P}$-game is the d-relaxed colouring game. We consider the $\mathcal{P}$-game when $\mathcal{P}$ is one of the following additive hereditary properties:
$\mathcal{O}_k = \{G : |V(G)| \leq k + 1\}$,
$\mathcal{S}_k = \{G : \Delta(G) \leq k\}$,
$\mathcal{D}_k = \{G : G \text{ is } k\text{-degenerated graph}\}$.
We discuss relations between the $\mathcal{P}$-game, the marking game and the edge version of the d-relaxed colouring game.

The least d such that Alice has a winning strategy on G for the d-relaxed colouring game with r colours is called the relaxed game defect of G and is denoted by $\mathrm{def}_g(G, r)$. By $\mathrm{def}_g^{\mathcal{O}}(G, r)$ we denote the least k such that Alice has a winning strategy on G for the $\mathcal{O}_k$-game with r colours. We determine the numbers $\mathrm{def}_g(G, r)$ and $\mathrm{def}_g^{\mathcal{O}}(G, r)$ for some classes of graphs.