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Adaptive Matching on Bipartite Graphs

We consider bipartite matching in the on-line version as follows. There is a bipartite graph $G = (U, V, E)$, in which vertices in V are given a priori and each vertex u in U arrives in the on-line fashion (with all its incident edges). An on-line algorithm matches each such vertex u to a previously unmatched adjacent vertex in V, if there is one. Decisions made by the algorithm are irrevocable. The objective is to maximize the size of the resulting matching. It is easy to observe that any greedy algorithm (never leave vertex u unmatched if a match is possible) matches at least $n/2$ edges where n is the size of the optimal matching with G given at once. This number is optimal and there is no better deterministic algorithm.

We propose the following deterministic modification of an on-line matching. The algorithm matches each incoming vertex u in U to a set $S(u)$ of adjacent vertices in V (instead of one vertex). In case when $S(u)$ and $S(x)$ for already existing x in U are not disjoint the algorithm must remove all common vertices from $S(x)$. Additionally, the algorithm has to obey the rule: each $S(x)$ must not become empty if only it was initialized with a nonempty set of vertices. An algorithm satisfying the above condition is called adaptive. In this approach a vertex u in U can be always matched to a vertex from $S(u)$ (if $S(u)$ is not empty). Therefore, the number of matched edges is equal to the number of nonempty sets $S(u)$. We are going to present the optimal adaptive on-line algorithm which breaks $n/2$ barrier and matches at least $(1 - \pi/\cosh(\sqrt{3}\pi/2))n + o(n) = 0.589n + o(n)$ edges.

This is joint work with Jakub Kozik.