Jolanta Rosiak

Local Exponents of Primitive Digraphs

A digraph D is primitive if there exists an integer $t > 0$ such that for all ordered pairs of vertices u and v (not necessarily distinct) there is a directed walk from u to v of length t in D. The smallest such t is called the exponent of D, denoted $\exp(D)$. If D is primitive and $u \in V(D)$, then there exists an integer $t > 0$ such that for each $v \in V(D)$ there exists a directed walk from u do v in D of length t. The smallest such t, denoted $\exp_D(u)$, is called the exponent of the vertex u in D (or the local exponent of D at the vertex u). Then $\exp(D) = \max_{u \in V(D)} \{\exp_D(u)\}$.

If D is a primitive digraph and $|V(D)| = n$, then the vertices can be relabeled as $v_1, v_2, \ldots, v_n$ so that $\exp_D(v_1) \leq \ldots \leq \exp_D(v_n)$. Let $\varepsilon(D) = \exp_D(v_n) - \exp_D(v_1)$. The problem of characterizing the sets $L^o_{n,m}(k) = \{\exp(D) : D \in \mathcal{DP}^o(n,m) \wedge \varepsilon(D) = k\}$, where $\mathcal{DP}^o(n,m)$ is the class of primitive digraphs with n vertices containing loops exactly at m vertices, is considered.

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