Maciej Zwierzchowski

On Signed Star Domination Number

All graphs considered here are finite, undirected without loops and multiple edges. For a vertex $v \in V(G)$, $E_G(v) = \{uv : u \in V(G)\}$ is called the *edge-neighbourhood* of v in G and $\deg_G(v) = |E_G(v)|$ is called a *degree* of v in G. A vertex $v \in V(G)$ is an *isolated* vertex of G if $\deg_G(v) = 0$. Additionally we assume that G has no isolated vertex.

A function $f : E(G) \to \{-1, 1\}$ is a *signed star domination function* (*SSDF*) of G if $\sum_{e \in E_G(v)} f(e) \geq 1$ for every $v \in V(G)$.

Let $E_f^+(G) = \{e \in E(G) : f(e) = 1\}$. A *SSDF* f is called a *minimal signed star domination function* (*mSSDF*) of G if for every edge $e \in E_f^+(G)$, a function $g : E(G) \to \{-1, 1\}$ such that

$$g(e') = \begin{cases} f(e') & \text{if } e' \neq e \\ -1 & \text{if } e' = e \end{cases}.$$

is not a SSDF of G.

For a vertex $v \in V(G)$ by a symbol $f(v)$ we mean $\sum_{e \in E_G(v)} f(e)$. A sequence $\pi = (w_1, \ldots, w_n)$ of positive integers is called a **signed star domination sequence** if there exists a graph G with $V(G) = \{v_1, \ldots, v_n\}$ and a minimal signed star domination function f of G such that $f(v_i) = w_i$. We present a necessary and sufficient conditions for a sequence π to be a signed star domination sequence.