Michał Dębski

Decomposing 2-colored Boolean Lattice into 2-colored Chains of Length 2

We consider the following problem: Let L be a finite boolean lattice. Each of its elements is colored either blue or red, with the following restrictions: (a) an element x is blue if and only if -x is red, and (b) if an element x is blue, than any element y such that y < x is also blue. The question is whether there exists a decomposition of L into blue-red pairs. Precisely: can lattice L be decomposed into disjoint chains of length 2, each consisting of a blue and a red element? This question was originally formulated by P. Mazur in terms of products of prime numbers.

We give the solution to this problem.