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On Rooted Packings of Graphs

A *vertex H-packing* (respectively an *edge H-packing*) into a graph G is a collection of vertex disjoint (resp. edge disjoint) subgraphs of G, each isomorphic to H. By a *rooted graph* we mean a pair (H, S), where H is a graph and $S \subseteq V(H)$. Rooted graphs (G, T) and (H, S) are *isomorphic* if there is a bijection $\varphi : V(G) \to V(H)$ such that $xy \in E(G)$ iff $\varphi(x)\varphi(y) \in E(H)$ and $\varphi(T) = S$. A collection of rooted graphs $\{(H_1, S_1), \dots, (H_k, S_k)\}$ isomorphic to (H, S) is a *rooted* (H, S)-*packing* into a graph G if $H_1, \dots, H_k$ are subgraphs of G, the edge sets $E(H_1), \dots, E(H_k)$ are pairwise disjoint and the vertex sets $S_1, \dots, S_k$ are pairwise disjoint. The concept of a rooted (H, S)-packing is a common generalization of both a vertex H-packing and an edge H-packing into a graph. We denote by $r(G, H, S)$ the largest number of copies of (H, S) in a rooted (H, S)-packing into G. A rooted (H, S)-packing into a graph G is a *rooted* (H, S)-*decomposition* (resp. a *rooted* (H, S)-*factor*) if $E(H_1) \cup \ldots \cup E(H_k) = E(G)$ (resp. $S_1 \cup \ldots \cup S_k = V(G)$). In the presented work we give a characterization of graphs that have a rooted (H, S)-decomposition and/or a rooted (H, S)-factor, for $H = K_{1,2}$ and some sets $S \subseteq V(H)$. In these cases we also compute the number $r(G, H, S)$. Next we give some negative computational complexity results on the problem of existence of a rooted (H, S)-decomposition, for some specific graphs H and sets $S \subseteq V(H)$.