

## Panagiotis-Georgios Tsikouras

### THE ENUMERATION OF STRINGS AT HEIGHT $j$ IN DYCK PATHS

This work deals with (lattice) paths in the integer plane, consisting of two steps, rises and falls. An important class of such paths are the Dyck paths, which start and end at the same height and lie weakly above this height. A wide range of papers dealing with the occurrences of a given path  $\tau$ , called in this context *string*, in a Dyck path appear frequently in the literature. All of them deal with particular strings, whereas recent results by the authors concerning arbitrary strings have been announced in the 7th Lattice Path Combinatorics Conference (2010).

A related problem is the enumeration of Dyck paths according to the occurrences of  $\tau$  at height  $j$ .

This problem has been studied for particular strings when  $j = 0$ , by many authors. For positive  $j$  and  $\tau = ud$  or  $\tau = du$ , Mansour (2002) expressed the generating function  $F_j$  which counts the number of occurrences of  $\tau$  at height  $j$  via the Chebyshev polynomials of the second kind and the Catalan generating function. This idea was extended by A. Sapounakis, I. Tasoulas and the author (2007) for an arbitrary string  $\tau$ , where  $F_j$  is expressed via the Chebyshev polynomials of the second kind and the generating function  $F_0$ . This result was then applied to every string of length 4, by evaluating  $F_0$  (and hence  $F_j$ , too).

In this work, for an arbitrary string  $\tau$  the generating function  $F_j$  is evaluated directly using a different method, based on the main characteristics of  $\tau$ , namely number of rises, height, depth and periodicity.

This is joint work with K. Manes, A. Sapounakis and I. Tasoulas.