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NONREPETITIVE SEQUENCES

A repetition of size h ($h \geq 1$) in a given sequence is a subsequence of consecutive terms of the form: $xx = x_1, \ldots, x_h, x_1, \ldots, x_h$. A sequence is nonrepetitive if it does not contain a repetition of any size. The remarkable construction of Thue asserts that 3 different symbols are enough to build an arbitrarily long nonrepetitive sequence. A sequence $s_1, \ldots, s_n$ is chosen from lists (sets) $L_1, \ldots, L_n$ if $s_i \in L_i$ for all i. It is still not settled what is the minimum size of all the lists required to ensure that there is a nonrepetitive sequence chosen from them. We present a very simple proof that lists of size 4 suffice (equalizing the best known bound) and provide a natural randomized algorithm constructing such a sequence in expected linear time. Our approach is inspired by a new algorithmic proof of Lovász local lemma due to Moser and Tardos and previous work of Moser (his so called entropy compression argument). We also consider game-theoretic versions of results on nonrepetitive sequences. A nonrepetitive game is played by two players who pick, one by one, consecutive terms of a sequence over a given set of symbols. First player tries to avoid repetitions while second one does not need to cooperate. Of course, by simple imitating the second player can force lots of repetitions of size 1. Pegden proved that there is a strategy for the first player to build an arbitrarily long sequence with no repetitions of size greater than 1 over 37 symbols. Our techniques allow to reduce 37 to 6. The second game we consider is an erase-repetition game. Here, whenever a repetition occurs the repeated block is immediately erased and the next player to play continues. We prove that there is a strategy for first player to build an arbitrarily long nonrepetitive sequence over 8 symbols.

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