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Small Subgraphs in Random Intersection Digraphs

Given subsets $S^{-}(1), \ldots, S^{-}(n)$ and $S^{+}(1), \ldots, S^{+}(n)$ of a set $W = \{w_1, \ldots, w_m\}$, define an intersection digraph on the vertex set $V = \{v_1, \ldots, v_n\}$ by putting an arc $v_i v_j$ for $i \neq j$ if and only if the sets $S^{-}(v_i)$ and $S^{+}(v_j)$ share a common element. Assuming the sets $S^{-}(i), S^{+}(i)$, $i = 1, \ldots, n$ are drawn at random, we obtain a random intersection digraph. Assuming further that for any $i = 1, \ldots, n$ and $j = 1, \ldots, m$ we have $P(w_j \in S^{-}(i)) = p_{-}(n, m)$ and $P(w_j \in S^{+}(i)) = p_{+}(n, m)$ and each inclusion is independent we obtain a binomial random intersection digraph $G(n, m, p_{-}, p_{+})$.

This (and more general) random intersection digraph model first studied by Bloznelis [1] is a natural extension of the random intersection graph introduced by Karoński, Scheinerman and Singer-Cohen [3]. The latter paper considers the problem of appearance of constant-size induced subgraphs H in the binomial random intersection graph $G(n, m, p)$ and obtains the explicit thresholds for appearance and disappearance for various types of small induced subgraphs H such as $H = K_h$, the complete graph on h vertices.

In the present work we consider appearance thresholds for small subgraphs of the binomial random intersection digraph $G(n, m, p_{-}, p_{+})$ and determine explicit functions that characterize the appearance threshold for $\overrightarrow{K}_h$, the digraph on h vertices with all possible arcs but without loops.

Identify each $w \in W$ with a unique colour. Then $G(n, m, p_{-}, p_{+})$ can be treated as a random coloured directed multigraph. We obtain that in almost all cases one of four simple types of coloured digraphs $\overrightarrow{K}_h$ is born first ("monochromatic", "rainbow", "in-star" or "out-star") depending on the parameters m, p_{-} and p_{+}.

References

[1] M. Bloznelis, *A Random Intersection Digraph: Indegree and Outdegree Distributions*, Discr. Math. 310, 19, 2010, 2560–2566

[2] E. Godehardt, J. Jaworski, *Two Models of Random Intersection Graphs for Classification*, Studies in Classification, Data Analysis and Knowledge Organization, Springer, Berlin, Heidelberg, New York, 2003, 67–81

[3] M. Karoński, E. R. Scheinerman, K. B. Singer-Cohen, *On Random Intersection Graphs: The Subgraph Problem*, Combinatorics, Probability and Computing, 8, 1999, 131–159